Whole cell tracking and movement reconstruction through an optimal control problem

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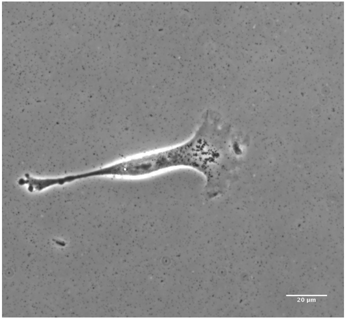
8 June 2015 at IBiDi

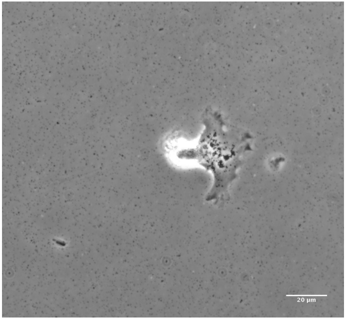
Outline

- Whole cell tracking through optimal control problem
  - The mathematical model
  - Toy models for proving concepts
  - Real world application
  - 3-D simulation
- IBiDi data
  - Our automatic segmentation algorithm
  - Particle tracking results

Objectives

To track the morphology of cells and reconstruct their movements:





V. Peschetola et al. *Cytoskeleton*, 2013

What is our signature

Particle tracking:

- The morphology of cells are not considered
- Manually tracking is slow
- Automatic tracking algorithms are often flawed
  - Segmentation is suboptimal for real data
  - Tracking through patten recognition is challenging

Pure geometric math models:

- Resolution of the data matters
- Typically no cell-setting is considered
- It is a complicated procedure to obtain the results
- Computational power and advanced numerical methods have to be included for 3-D real-life cell tracking

Our optimal control model

The volume conserved mean curvature flow:

$$\begin{cases} \boldsymbol{V}(\boldsymbol{x},t) & = (-\sigma H(\boldsymbol{x},t) + \eta(\boldsymbol{x},t) + \lambda_V(t))\,\boldsymbol{v}(\boldsymbol{x},t) \ \text{ on } \ \Gamma(t), t \in (0,T], \\ \Gamma(0) = \Gamma^0. \end{cases}$$

The phase-field approximation of the above equation - Allen-Cahn:

$$\begin{cases} \partial_t \phi(\boldsymbol{x},t) & = \triangle\phi(\boldsymbol{x},t) - \frac{1}{\epsilon^2} G'(\phi(\boldsymbol{x},t)) - \frac{1}{\epsilon}(c_G \eta(\boldsymbol{x},t) - \lambda(t)) \ \text{ in } \ \Omega \times (0,T], \\ \nabla\phi \cdot \boldsymbol{\nu}_\Omega & = 0 \ \text{ on } \ \partial\Omega \times (0,T], \\ \phi(\cdot,0) & = \phi^0 \ \text{ in } \ \Omega. \end{cases}$$

Our optimal control model cont.

The objective functional:

$$J(\phi, \eta) = \frac{1}{2} \int_{\Omega} \left(\phi(\boldsymbol{x}, T) - \phi_{obs}(\boldsymbol{x})\right)^2 d\boldsymbol{x} + \frac{\theta}{2} \int_0^T \int_{\Omega} \eta(\boldsymbol{x}, t)^2 d\boldsymbol{x} dt,$$

and now we solve the minimisation problem:

$$min_{\eta} J(\phi, \eta), \quad \text{with } J \text{ given above.}$$

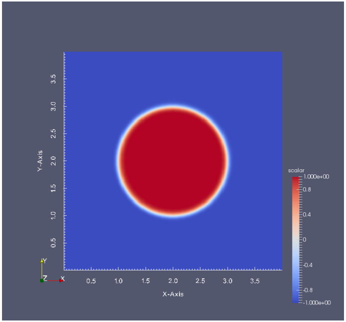
Obtaining solutions

We are using one of the most efficient solution methods, combining most advanced adaptive techniques. Meanwhile, parallelism is employed and the computation has been carried out on large computer cluster with multiple number of computational cores.

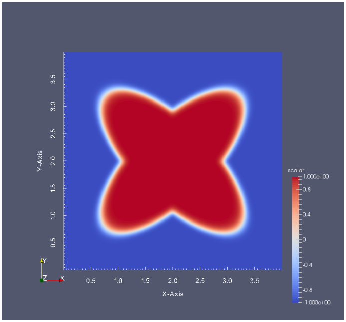
Toy model for proving concepts

A circle becomes two ellipses.

Initial data



Desired data

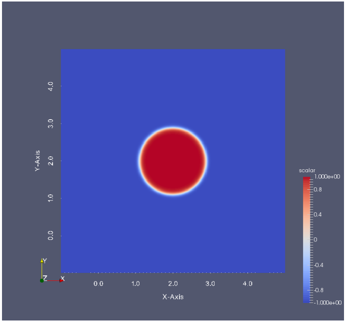


Toy model for proving concepts

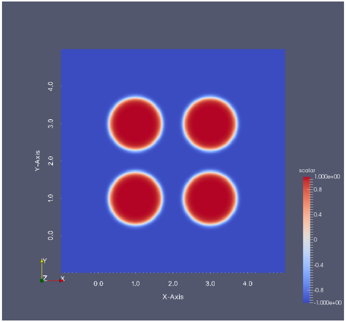
Toy model for proving concepts cont.

A circle becomes 4 children circles.

Initial data



Desired data



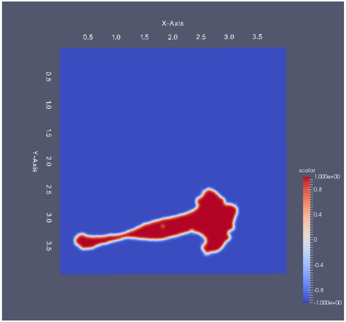
Toy model for proving concepts cont.

Toy model for proving concepts cont.

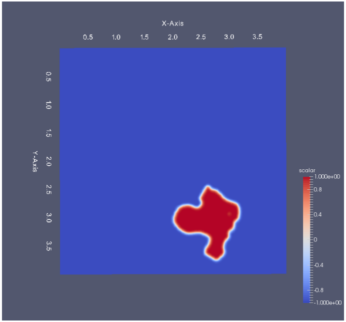
Real world application

Two segmented cell images.

Initial data



Desired data

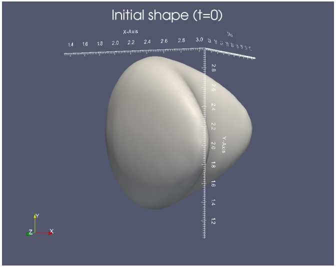


Real world application cont.

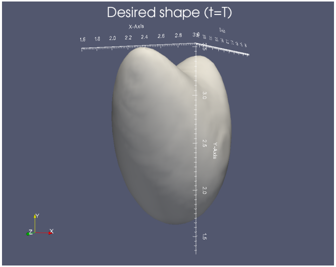
Real world application cont.

3-D simulation

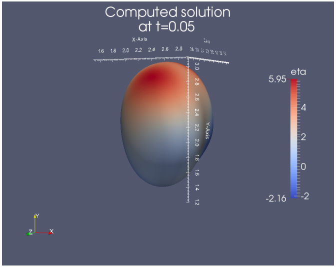
(a)



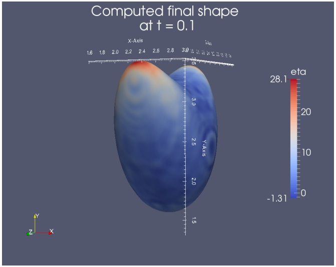
(b)



(c)



(d)



3-D simulation cont.

IBiDi data

IBiDi data cont.

Issues with basic segmentation

Our simple solution to segmentation

Particle tracking

N. Stuurman. *Mtrack2 at valelab.ucsf.edu*, 2009

Particle tracking cont.

The end

Thank you.